**In-Lab Group Activity for Week 6: Linear Transformations, Block Matrices**

**Name: Cole Bardin**

***first     last***

**Question 1: Matrix of a Linear Transformation**

Consider a linear transformation that maps vectors from to vectors in .

We are given the action of *T* on the standard basis for

, where and

**a.** What are the dimensions of the matrix *A* for *T* in the standard basis?

**i.** **ii.** **iii.** i**v.**

**b.** Find the matrix *A* for *T* in the standard basis such that for any vector in , T(

**c.** Find the image of the vector under the linear transformation using your matrix *A*.

**d.** Row reduce your matrix *A* to find the rank of *T*. (**Hint**: Same as the rank of its matrix *A*).

3

So, the rank of *A* (and *T*) is:

0

**e.** What is the **nullity** of *T*? **Hint**: Same as the number of free variables.

**Question 2: Matrix of a new Linear Transformation *S* with Missing Information**

Consider a new linear transformation that also maps vectors from to vectors in .

Unfortunately, we do not (yet) know its action on a complete **basis**, but only for the first two standard basis vectors.

where .

We are told nothing about its action on the third basis vector . It could be anything.

**a.** Select the form which includes every matrix *A* which might represent *S*, the unknown values of which will depend on . We can denote an arbitrary vector in as .

**i.** **ii.**  **iii.**  **iv.**

**b.** Find the image of the vector under the linear transformation.

5\*e1+3\*e2

**i.** First write as a linear combination of the vectors and , so that

**ii.** Now apply the linearity of *S* to find :

**c.** What is ?

**d.** For which of these can you **not** find its image under *S* without knowing the additional information for ?

**i.** **ii.**  **iii.** **iv.** **v.** We can't find any of these.

**Wait!** We were just handed some new info about *S*! Only use this new info in the parts below!

We are now given where . Note that is not one of the standard basis vectors!

**e.** Find the image of the third standard basis vector under *S*.

**i.** First you must write in terms of the given vectors . You can do this in your head.

**ii.** Now use your expansion (and linearity) to find:

**f.** Give the matrix *A* for *S* in the standard basis using the new info. Now you know all three columns!

**g.** Using your matrix *A*, confirm where we recall . Just multiply *A* times .

**Question 3: Block Matrices:** Find the inverse of the block matrix *A* where are symbolic variables. Note block forms are treated in Lecture 7, but you can do it now.

**This is a block decomposition of the matrix *A*.**

**a.** Write out each of the block matrices below.Some blocks are partially filled for free.

,     ,     ,

**b.** Which of these block matrices are their own inverse? That's unusual but was designed to make your work easier. **Hint**: Just check if .

**i.** Only  **ii.** Only **iii.** Both and  **iii.** Neither nor

**c.** In lectures, we will derive a formula for the inverse of a block upper triangular matrix.

Recall if is invertible with and both square, then the diagonal blocks and are also invertible; and the inverse of the entire matrix *A* is:

where we are **also** using *B* to denote the inverse. Compute by explicitly performing the matrix products:

**d.** Notice that **magically,** for this special matrix, . Thus, the inverse of the entire matrix *A* is:

**i. ii.**  **iii.** the identity matrix  **iv.** None of these